CONVECTIVE STABILITY OF A MEDIUM CONTAINING A HEAVY SOLID ADDITIVE

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Two problems of convective stability in a medium containing settling heavy solid particles are discussed. A study is made of the stability of steady convective flow of a medium containing an additive between vertical plates heated to different temperatures and also of the stability of a flat layer of a medium containing an additive which is heated from below. It is shown that the presence of settling solid particles has a significant stabilizing effect on convective stability.

The stability of isothermal plane-parallel flows of an incompressible gas transporting a small amount of solid particles was studied in [1-4]. The transporting medium and the additive were considered as interpenetrating and interacting continuous media; interaction between particles was neglected. A formulation of the problem of flow stability based on these concepts was first given in [1] where stability of motion in a plane vertical channel was considered for a fluid containing an additive. The stability of convective motion of a medium transporting a solid additive in a layer between vertical plates heated to different temperatures was studied in [5] where the settling of the particles was neglected, as was the case in [2-4].

The effect of suspended solid particles on the equilibrium stability of a horizontal layer of a gas heated from below was considered in [6]. Particle settling and the displacement force acting on the particles were neglected. The existence of thermal equilibrium between particles and gas was assumed, i.e., the simple limiting case of an infinitely short temperature relaxation time τ_T was considered. Under the assumptions described, the effect of particles present in a layer reduces to a mere renormalization of the heat capacity of the gas and so to a trivial renormalization of the Rayleigh number also.

In the following, a study is made of the effect on convective stability of all factors characterizing the added particles: the rate of particle settling u_s, the velocity and temperature relaxation times for the particles (or, which comes to the same thing, their size, density, and heat capacity), and the mass concentration a of the additive.

§ 1. We consider an incompressible fluid containing a cloud of spherical nondeformable particles of identical mass m and radius r. The density ρ_1 of the particle material is much greater than the density ρ of the transporting medium. The volumetric particle fraction is $f \ll 1$ and therefore interactions between particles can be neglected. The mass concentration a of the particles is not assumed small and can reach a value of 0.2. In this case, one cannot consider the Einstein correction to the viscosity of a fluid, which is proportional to the volumetric concentration f of an additive. The displacement force acting on a particle is negligibly small since it is proportional to the ratio ρ/ρ_1. The particles are large enough to exclude participation in Brownian motion; there is no pressure associated with the particle cloud. Interaction force between phases during their relative motion is described by Stokes' law.

Equations describing the behavior of a medium containing a cloud of solid particles were given in [7, 8]. Based on those equations, equations were obtained [5] in the Boussinesq approximation [9] for the free convection of an incompressible medium with a heavy additive:

$$\partial \mathbf{u}/\partial t + (\mathbf{u}\nabla)\mathbf{u} = -\nabla p/\rho + \nu\Delta\mathbf{u} + a/\tau_v(\mathbf{u}_p - \mathbf{u}) - (1+a)\mathbf{g}\beta T, \quad (1.1)$$
$$\partial \mathbf{u}_p/\partial t + ((\mathbf{u}_p + \mathbf{u}_s)\nabla)\mathbf{u}_p = -(1/\tau_v)(\mathbf{u}_p - \mathbf{u}),$$

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$$\partial T/\partial t + \mathbf{u}\nabla T = \chi\Delta T + (ab/\tau_T)(T_p - T),$$
$$\partial T_p/\partial t + (\mathbf{u}_p + \mathbf{u}_s)\nabla T_p = -(1/\tau_T)(T_p - T),$$
$$\operatorname{div}\mathbf{u} = 0,\ \partial N/\partial t + \operatorname{div}[N(\mathbf{u}_p + \mathbf{u}_s)] = 0,$$
$$\tau_\nu = m/6\pi r\nu\rho,\ \ \tau_T = mb/4\pi r\chi\rho,\ \mathbf{u}_s = mg/6\pi r\nu\rho,$$
$$a = \rho_p/\rho,\ b = C_1/C,\ \rho_p = Nm,$$

where u is velocity; T is temperature; p is the pressure of the fluid measured with respect to the hydrostatic pressure renormalized because of the presence of settling particles; c is the heat capacity of the fluid at constant pressure; β, ν, and χ are the coefficient of volume expansion of the fluid, its kinematic viscosity, and thermal diffusivity; g is the acceleration of free fall. Quantities with the subscript p refer to the cloud of particles, where $\mathbf{u}_p$ is the velocity acquired by the particles as a result of their interaction with the moving fluid measured with respect to the rate of particle settling $\mathbf{u}_s$; c_1 is the heat capacity of the particle material; N is the number of particles per unit volume.

The quantities τ_T and τ_V have the dimensionality of time and are, respectively, the time required for the temperature difference between fluid and particles to decrease by a factor e and the time required for the velocity of the particles relative to the fluid to decrease by a factor of e in comparison with its original value.

§ 2. We consider convective motion of a fluid containing an additive in a plane layer between infinite parallel vertical surfaces at $x = \pm h$, which are maintained at the constant temperatures $-\Theta$ and Θ, respectively. The particles, the concentration of which is uniform, move through the fluid.

We obtain a steady-state solution of the equation system (1.1) describing plane-parallel convective motion in such a structure,

$$u_x = u_y = 0,\ \ u_z = u_0(x),\ \ T_0 = T_0(x),\ \ p_0 = p_0(z),$$
$$u_{px} = u_{py} = 0,\ \ u_{pz} = u_{p0}(x),\ \ T_{p0} = T_{p0}(x),\ \ N_0 = \text{const} \tag{2.1}$$

[the subscript 0 denotes a steady-state solution of the system (1.1)].

Using Eqs. (2.1), we obtain from (1.1) the system of equations

$$\nu d^2u_0/dx^2 + (1 + a)g\beta T_0 = (1/\rho)dp_0/dz = c,\ \ u_{p0} = u_0; \tag{2.2}$$
$$d^2T_0/dx^2 = 0,\ T_{p0} = T_0, \tag{2.3}$$

where c is the constant of separation of variables. To determine u_0, T_0, and p_0, we used the boundary conditions

$$u_0(\pm h) = 0,\ T_0(\pm h) = \mp\Theta \tag{2.4}$$

and the closure condition for convective flow

$$\int_{-h}^{h} u_0 dx = 0. \tag{2.5}$$

We obtain from Eqs. (2.2)-(2.5) the distributions of velocities and temperatures of the fluid and particle cloud over a section of the layer:

$$u_0 = (1 + a)(g\beta\Theta h^2/6\nu)(x^3/h^3 - x/h),$$
$$U_{p0} = u_0 + u_s,\ T_0 = T_{p0} = -(\Theta/h)x. \tag{2.6}$$

As is clear from Eqs. (2.6), the presence of added particles leads to renormalization of the velocity profile of the fluid in comparison with the case of a fluid without an additive [9].

§ 3. We investigate the stability of the steady-state motion of a medium containing a heavy additive as defined by Eqs. (2.6). To do this, we consider the perturbed fields for velocity, temperature, pressure, and number of particles per unit volume, $\mathbf{u}_0 + \mathbf{u}$, $T_0 + T$, $\mathbf{U}_{p0} + \mathbf{u}_p$, $T_{p0} + T_p$, $p_0 + p$, and $N_0 + N$, where $\mathbf{u}$, $\mathbf{u}_p$, T, T_p, p, and N are small perturbations.

We write the equations for the perturbations in dimensionless form, using the following units of measurement: distance h, time h^2/ν, velocity ν/h, pressure $\rho\nu^2/h^2$, and temperature Θ. Linearizing over the perturbations, we obtain from Eqs. (1.1)

$$
\begin{aligned}
&\partial \mathbf{u}/\partial t + (\mathbf{u}\nabla)\mathbf{u}_0 + (\mathbf{u}_0\nabla)\mathbf{u} = -\nabla p + \nabla \mathbf{u} + \boldsymbol{\gamma}\mathrm{Gr}\,T + (a/\tau_v)(\mathbf{u}_p - \mathbf{u});\\
&\partial \mathbf{u}_p/\partial t + (\mathbf{u}_p\nabla)\mathbf{u}_0 + ((\mathbf{u}_0 + \mathbf{u}_s)\nabla)\mathbf{u}_p = -(1/\tau_v)(\mathbf{u}_p - \mathbf{u});\\
&\partial T/\partial t + \mathbf{u}\nabla T_0 + \mathbf{u}_0\nabla T = (1/\mathrm{Pr})\nabla T + (ab/\tau_T)(T_p - T);\\
&\partial T_p/\partial t + \mathbf{u}_p\nabla T_0 + (\mathbf{u}_0 + \mathbf{u}_s)\nabla T_p = -(1/\tau_T)(T_p - T);\\
&\mathrm{div}\,\mathbf{u} = 0;\ \partial N/\partial t + \mathrm{div}[N(\mathbf{u}_0 + \mathbf{u}_s) + N_0\mathbf{u}_p] = 0,\\
&\mathbf{u}_s = -\mathrm{Ga}\tau_v\boldsymbol{\gamma},\ \mathbf{u}_0 = (\mathrm{Gr}/6)(x^3 - x)\boldsymbol{\gamma},\ T_0 = -x,\\
&\tau_v = (2/9)(r/h)^2(\rho_1/\rho),\ \tau_T = (3/2)b\mathrm{Pr}\tau_v,\\
&\mathrm{Ga} = gh^3/\nu^2,\ \mathrm{Gr} = (1 + a)g\beta\Theta h^3/\nu^2,\ \mathrm{Pr} = \nu/\chi,
\end{aligned}
\tag{3.1}
$$

where Ga, Gr, and Pr are the Galileo, Grashof, and Prandtl numbers; τ_V and τ_T are now dimensionless relaxation times; γ is a unit vector directed vertically upward.

As in the case of a pure fluid [9, 10], one can show for a medium containing an additive that the problem of stability with respect to spatial perturbations reduces to the corresponding problem for plane perturbations. Plane perturbations are more dangerous in the case of vertical orientation of the layer, i.e., lower Grashof numbers are associated with them. Consequently, it is sufficient to confine the investigation to plane perturbations in a study of stability.

We consider plane normal perturbations

$$
\begin{aligned}
&u_x = -\partial\psi/\partial z,\ u_z = \partial\psi/\partial x,\\
&\psi(x, z, t) = \varphi(x)\exp[ik(z - ct)],\ T(x, z, t) = \theta(x)\exp[ik(z - ct)],\\
&u_{px}(x, z, t) = v_{px}(x)\exp[ik(z - ct)],\ u_{pz}(x, z, t) = v_{pz}(x)\exp[ik(z - ct)],
\end{aligned}
\tag{3.2}
$$

where ψ is a stream function; φ, θ, v_{px}, and v_{pz} are the amplitudes of the perturbations; k is a real wave number; $c = c_r + ic_i$ is the complex phase velocity of the perturbations (c_r is the phase velocity, c_i the decrement).

Substituting Eqs. (3.2) into Eqs. (3.1), we obtain a system of amplitude equations (primes denote differentiation with respect to x)

$$
\begin{aligned}
&(\varphi^{IV} - 2k^2\varphi'' + k^4\varphi) - ik(\varphi'' - k^2\varphi)(u_1 - c) + \mathrm{Gr}\,\theta' + ik\varphi u_1'' = 0;\\
&\frac{1}{\mathrm{Pr}}(\theta'' - k^2\theta) - ik\theta(u_2 - c) + ik\varphi T_0' A = 0,
\end{aligned}
\tag{3.3}
$$

where

$$
\begin{aligned}
&u_1 = u_0 + a(u_0 + u_s - c)/[1 + ik\tau_v(u_0 + u_s - c)];\\
&u_2 = u_0 + ab(u_0 + u_s - c)/[1 + ik\tau_T(u_0 + u_s - c)];\\
&A = 1 + ab/\{[1 + ik\tau_v(u_0 + u_s - c)][1 + ik\tau_T(u_0 + u_s - c)]\}.
\end{aligned}
$$

Boundary conditions are

$$
\varphi = \varphi' = \theta = 0 \quad \text{for} \quad x = \pm 1. \tag{3.4}
$$

The boundary-value problem (3.3), (3.4) determines the spectrum of characteristic perturbations and their decrements. The complex phase velocity c depends on seven independent parameters of the problem: the Grashof, Prandtl, and Galileo numbers; the wave number k; the mass concentration a of the additive; and the relaxation times τ_T and τ_V. The limit of stability for steady-state flow is determined from the condition $c_i = 0$.

To solve the resultant boundary-value problem, i.e., to determine the decrement spectrum and the flow stability limits, the Runge–Kutta–Merson method of stepwise integration was used with orthogonalization of solutions at each step in the integration [11, 12]. The method used made it possible to carry out calculations to sufficiently large values of the problem parameters: $\mathrm{Gr} \sim 10^5$, $\mathrm{Pr} \sim 10^2$, $\mathrm{Ga} \sim 10^6$.

§ 4. Calculations performed over a broad range of values of the Prandtl number ($10^{-2} \leq \mathrm{Pr} \leq 10^2$) showed that steady-state motion of a medium containing an additive [Eqs. (2.6)] has two forms of instability. The first is associated with flow structure through the existence of two opposing flows, the interaction between which leads to loss of stability. The second form of instability is produced by the buildup of thermal waves in the flow at sufficiently large Prandtl numbers $\mathrm{Pr} \geq \mathrm{Pr}_*$ ($\mathrm{Pr}_* \approx 11$).

For values of the Prandtl number less than the critical value, $\mathrm{Pr} < \mathrm{Pr}_*$, the instability of the steady-state motion of a fluid containing an additive is caused by the lowest modes of hydrodynamic perturbations

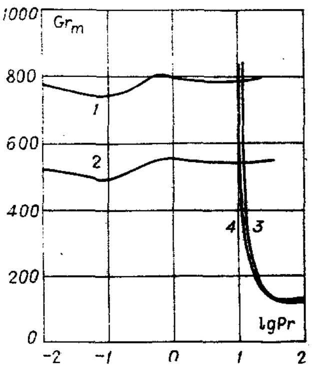

Fig. 1

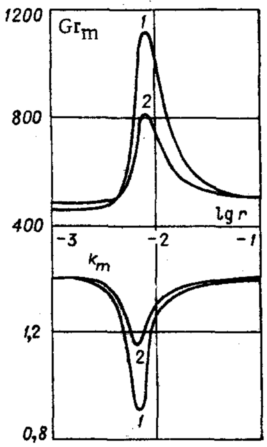

Fig. 2

(thermal perturbations become more dangerous when $Pr > Pr_*$). The effect of thermal factors on this form of instability is insignificant. Settling particles produce oscillational (traveling) perturbations and facilitate their transport. Consequently, the instability associated with the lowest hydrodynamic modes of the perturbations is oscillational. The instability of steady-state motion of a pure fluid for $Pr < 11.4$ is associated with monotonic perturbations [9, 13] just as in the case of a fluid containing an additive with a settling rate that can be neglected in comparison with the velocity of steady-state flow of the fluid [5]. Since the instability mechanism is the same in all three cases, this form of instability of the motion of a medium containing an additive can be called quasimonotonic.

We consider quasimonotonic instability of the steady-state motion (2.6). Its limit varies little for changes of the Prandtl number over a wide range ($10^{-2} \le Pr \le 30$). The critical stability region has a clearly expressed hydrodynamic nature.

Figure 1 shows the dependence of the minimum critical Grashof number Gr_m on the Prandtl number for the following values of problem parameters: $a = 0.05$, $Ga = 43{,}600$ ($b = 2.7$, $\rho_1/\rho = 415$). Curve 1 corresponds to $\tau_v = 0.0049$ ($r/h = 0.0073$) and curve 2 to $\tau_v = 0.00083$ ($r/h = 0.003$). It is clear that an increase in particle size leads to considerable stabilization of flow (curve 2 practically coincides with the corresponding curve for a pure fluid [9]). The perturbation phase velocity c_{rm} ($c_r < 0$) corresponding to the minimum critical Grashof number Gr_m also changes insignificantly as Pr varies ($c_{rm} \approx -5.25$). The critical wave number k_m corresponding to Gr_m depends slightly on Prandtl number ($k_m \approx 1.15$).

The dependence of the minimum critical Grashof number Gr_m on the mass concentration a of the additive turns out to be linear. The number of particles per unit volume increases with an increase in a for $\tau_v = \text{const}$ ($r = \text{const}$, $\rho_1/\rho = \text{const}$), i.e., the influence of the additive on flow stability grows. The minimum critical Grashof number Gr_m increases from 500 to 1050 with an increase in the mass concentration a from 0 to 0.1; the critical phase velocity c_{rm} falls from 0 to −12 and k_m decreases linearly from 1.42 to 1.00 ($Ga = 43{,}600$, $Pr = 0.73$, $\tau_v = 0.0049$, $\tau_T = 0.0145$). The additive facilitates dissipation of perturbation energy in some frequency range during the interaction of inert particles with velocity pulsations. Long-wave perturbations become responsible for the critical region of flow.

Figure 2 shows the dependence of the minimum critical Grashof number Gr_m and of the critical wave number k_m on the radius r of the added particles (r denotes the dimensionless radius of the particles) for two values of the mass concentration a of the additive. Curve 1 corresponds to $a = 0.1$ and curve 2 to $a = 0.05$ for $Ga = 43{,}600$, $Pr = 0.73$ ($b = 2.7$, $\rho_1/\rho = 415$). These values of problem parameters correspond to wood dust in air. An increase in the particle radius r for $a = \text{const}$ leads to a clearly expressed stabilization effect on flow up to a critical value $r_* \approx 0.0079$ after which the stabilizing effect decreases with increasing r. Here two opposing factors are competing [2]; increase in particle size leads to additional dissipation of perturbation energy, but the number of particles is reduced in this case ($a = \text{const}$, $\rho_1/\rho = \text{const}$), i.e., their influence on flow stability is weakened. The nature of the dependence of the critical wave number k_m on particle radius is evidence that stabilization of flow is produced through suppression of dangerous perturbations by the particles. The curves reflecting the dependence of the value of the critical phase velocity $|c_{rm}|$ of the perturbations on particle radius have a shape similar to the curve for $Gr_m = Gr_m(r)$ in Fig. 2. For $r = 0.0075$, the critical phase velocity has the minimum value $c_{rm} = -12.2$ ($a = 0.1$, $Ga = 43{,}600$, $Pr = 0.73$, $b = 2.7$, $\rho_1/\rho = 415$).

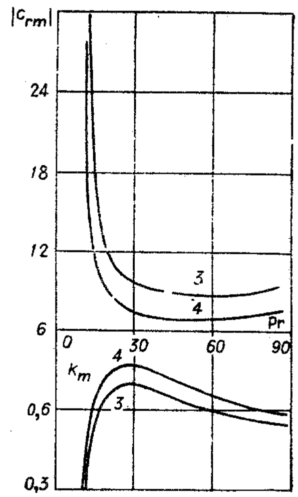

Fig. 3

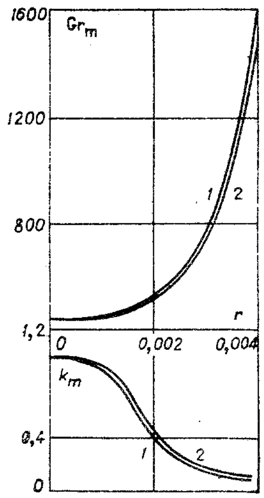

Fig. 4

§ 5. We consider oscillational instability of steady-state motion, i.e., an instability created by the lowest thermal modes of perturbations which are built up in the flow by thermal waves. In contrast to quasimonotonic instability, oscillational instability is essentially associated with the nonisothermal nature of the flow. In the case of a pure fluid, oscillational instability is produced by a pair of complex-conjugate decrements [9]. Thermal waves which are propagated both upward and downward in the flow with identical absolute values of the phase velocity are equally possible. Settling solid particles, which produce perturbations that travel downward along the layer, make that direction most favorable for the propagation of perturbations. Now the perturbations traveling downward have a greater absolute value of the phase velocity than the perturbations traveling upward. Inclusion of particle settling leads to the removal of the degeneracy of thermal decrements. In such a case, one can speak of two neutral curves of oscillational instability corresponding to a pair of the lowest thermal decrements.

As in the case of a pure fluid [9, 13], a shift in the form of instability occurs when $Pr > Pr_*$ ($Pr_* \approx 11$) with the oscillational perturbations becoming most dangerous.

Figure 1 shows the dependence of the minimum critical Grashof number Gr_m on the Prandtl number Pr (instability with respect to oscillational perturbations). The problem parameters are the following: $a = 0.05$, $Ga = 43{,}600$, $\tau_v = 0.00021$ ($b = 2.7$, $r = 0.0015$, $\rho_1/\rho = 415$, $u_s = -9.05$). Curve 3 corresponds to a perturbation phase velocity $c_r < 0$ and curve 4 to $c_r > 0$. As the Prandtl number increases, flow stability with respect to oscillational perturbations decreases up to $Pr \approx 57$ and then begins to increase. Up to Prandtl numbers $Pr \approx 40$, perturbations traveling upward along the layer are most dangerous, and perturbations traveling downward become more dangerous when $Pr > 40$.

Figure 3 shows the dependence on Prandtl number for the critical wave number k_m and for the absolute value of the phase velocity c_{rm}. The problem parameter values correspond to those for curves 3 and 4 in Fig. 1.

The dependence of the critical numbers k_m and Gr_m on particle radius r is shown in Fig. 4. Curve 1 corresponds to negative perturbation phase velocity (decrement ν_1) and curve 2 to positive phase velocity (decrement ν_0). Values of the problem parameters are as follows: $a = 0.05$, $Ga = 43{,}600$, $Pr = 30$ ($\rho_1/\rho = 415$, $b = 2.7$). The stabilizing effect of the additive on the stability of steady-state motion of the fluid increases as particle size increases. The absolute values of the phase velocities of perturbations traveling both upward and downward along the layer increase rapidly with increase in r. It is clear from a comparison of Figs. 2 and 4 that the added particles suppress thermal perturbations considerably more effectively. The stability of steady-state convective motion of a fluid with respect to quasimonotonic perturbations can be increased by a factor of 2–2.5 by adding heavy particles ($a \le 0.2$) to the flow and by a factor of more than 10 with respect to oscillational perturbations.

A comparison of these results with the results of [5] shows that settling particles produce a considerably greater stabilizing effect on steady-state flow of a fluid than suspended particles. In fact, neglect of particle settling rate in comparison with the velocity of steady-state flow of a fluid is only valid for sufficiently fine particles of not too great a density (with respect to the density of the transporting medium). Coarse dense

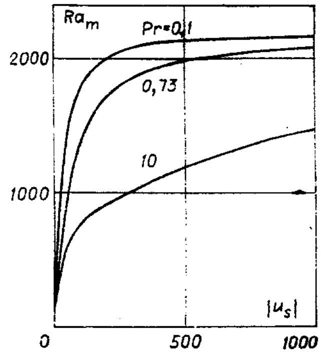

Fig. 5

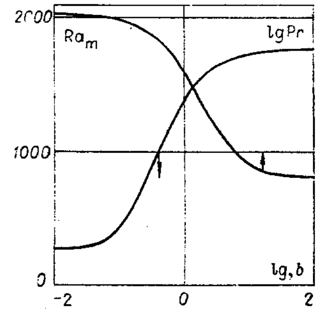

Fig. 6

particles are more inert than fine particles, and it is impossible to neglect their settling rate. The particle slip rate with respect to the fluid is of the order of the quantity u_s. The resultant relative motion of fluid and particles leads to additional dissipation of perturbation energy in comparison with the case of suspended particles.

§ 6. We consider a horizontal plane layer of incompressible fluid or gas bounded by infinite solid surfaces at $z = \pm 1$. The layer is heated from below. Particles, the concentration of which is uniform ($N_0 = \text{const}$), enter the layer through the upper surface. The particles settle and therefore there is transverse motion of the additive with a uniform vertical velocity u_s in the unperturbed state in the layer. We assume that heating of particles at the lower surface does not occur. In fact, the volumetric concentration of the additive is $f \ll 1$, and the change in layer thickness because of settled particles is insignificant. One can also assume that the lower bounding surface is permeable for the particles.

We determine steady-state distributions of the temperatures T_0 of the gas and T_{p0} of the particle cloud in the absence of convective motion in this two-phase system [the subscript 0 now denotes a steady-state solution of the system (1.1) with $\mathbf{u}_0 = 0$]. To accomplish this, it is necessary to solve the dimensionless equations of thermal conductivity obtained from the appropriate equations in the system (1.1) written in dimensionless form with $\mathbf{u}_0 = 0$ (in this case, it is convenient to select χ/h and $\rho_0 \nu \chi/h^2$, respectively, as units of velocity and pressure),

$$T_0'' + \frac{ab\,\mathrm{Pr}}{\tau_T}(T_{p0} - T_0) = 0,\quad u_s T_{p0}' + \frac{\mathrm{Pr}}{\tau_T}(T_{p0} - T_0) = 0 \tag{6.1}$$

(the primes denote differentiation with respect to z).

Boundary conditions are

$$T_0 = \mp 1 \text{ for } z = \pm 1;\quad T_{p0} = -1 \text{ for } z = 1. \tag{6.2}$$

Particles enter the layer with a temperature equal to that of the upper boundary.

The temperature distributions in the gas layer and in the particle cloud during steady-state transverse motion of the additive have the form

$$\begin{aligned} T_0 &= a_1[\exp(k_1(z-1)) - 1] + a_2[\exp(k_2(z-1)) - 1] - 1; \\ T_{p0} &= a_1[(k_1/abu_s)\exp(k_1(z-1)) - 1] + a_2[(k_2/abu_s)\exp(k_2(z-1)) - 1] - 1, \end{aligned} \tag{6.3}$$

where

$$a_1 = 2/[1 - \exp(-2k_1)](k_3 - 1);\quad a_2 = 2/[1 - \exp(-2k_2)](1/k_3 - 1);$$

$$k_1 = -\frac{\mathrm{Pr}}{2\tau_T u_s} + \sqrt{\left(\frac{\mathrm{Pr}}{2\tau_T u_s}\right)^2 + \frac{\mathrm{Pr}\,ab}{\tau_T}};\quad k_2 = -\frac{\mathrm{Pr}}{2\tau_T u_s} - \sqrt{\left(\frac{\mathrm{Pr}}{2\tau_T u_s}\right)^2 + \frac{\mathrm{Pr}\,ab}{\tau_T}};$$

$$k_3 = [1 - \exp(-2k_2)]/[1 - \exp(-2k_1)] \cdot [(abu_s - k_1)/(abu_s - k_2)].$$

In the limiting case of suspended particles ($\mathbf{u}_s = 0$), we obtain from Eqs. (6.1) and (6.2) a vertically linear distribution of the temperatures, $T_{p0} = T_0 = -z$. As is clear from Eq. (6.3), the temperature distributions in the gas and particle cloud are different from linear when the particle settling rate is nonzero. The distortion of the linear distribution of gas temperature increases with an increase in the particle settling rate and also with an increase in the mass concentration and relative heat capacity of the particles. With further increase in the parameters listed, a tendency toward the formation of a boundary layer within which the main change in gas temperature is concentrated is noted at the lower boundary.

§ 7. To study the convective stability of an equilibrium layer of a medium containing settling particles, we consider the perturbed fields for velocity, temperature, pressure and particle number, **u**, $\mathbf{u}_p+\mathbf{u}_s$, T_0+T, $T_{p0}+T_p$, p_0+p, and N_0+N, where **u**, $\mathbf{u}_p$, T, T_p, p, and N are small perturbations. Equations for the perturbations can be obtained from (1.1) by linearization over the perturbations. Eliminating the pressure and the x, y components of the velocities of the gas and particle cloud from these equations in the usual manner, one can obtain equations for the vertical components of the perturbation velocities $u_z(x, y, z, t)$ and $u_{pz}(x, y, z, t)$ and for the temperatures $T(x, y, z, t)$ and $T_p(x, y, z, t)$. We consider normal perturbations of the form

$$\begin{aligned} u_z &= v(z)\exp[-\lambda t + i(k_1x + k_2y)]; \\ u_{pz} &= v_p(z)\exp[-\lambda t + i(k_1x + k_2y)]; \\ T &= \theta(z)\exp[-\lambda t + i(k_1x + k_2y)], \\ T_p &= \theta_p(z)\exp[-\lambda t + i(k_1x + k_2y)], \end{aligned} \tag{7.1}$$

where k_1 and k_2 are real wave numbers along the x and y directions; $\lambda=\lambda_r+i\lambda_i$ is the complex decrement of the perturbations. Taking into account the form (7.1) of the perturbations, we finally obtain dimensionless equations for the perturbation amplitudes

$$\begin{gathered} (v^{IV} - 2k^2v'' + k^4v) - \left(\frac{a}{\tau_v} - \lambda\right)(v'' - k^2v) - \mathrm{Ra}\,k^2\theta + \\ + \frac{a}{\tau_v}\left\{\frac{\mathrm{Pr}}{u_s\tau_v}\left[v' - \frac{\mathrm{Pr}}{u_s}\left(\frac{1}{\tau_v} - \lambda\right)v\right] + \left[\frac{\mathrm{Pr}^2}{u_s^2}\left(\frac{1}{\tau_v} - \lambda\right)^2 - k^2\right]v_p\right\} = 0; \\ \frac{u_s}{\mathrm{Pr}}v_p' + \left(\frac{1}{\tau_v} - \lambda\right)v_p - \frac{v}{\tau_v} = 0; \\ \frac{1}{\mathrm{Pr}}(\theta'' - k^2\theta) - \left(\frac{ab}{\tau_T} - \lambda\right)\theta - \frac{T_0'}{\mathrm{Pr}}v + \frac{ab}{\tau_T}\theta_p = 0; \\ \frac{u_s}{\mathrm{Pr}}\theta_p' + \left(\frac{1}{\tau_T} - \lambda\right)\theta_p + \frac{T_{p0}'}{\mathrm{Pr}}v_p - \frac{1}{\tau_T}\theta = 0; \\ \mathrm{Ra} = (1 + a)\frac{g\beta\theta h^3}{\nu\chi},\ u_s = -\tau_v\frac{\mathrm{Ga}}{\mathrm{Pr}}\boldsymbol{\gamma},\ k^2 = k_1^2 + k_2^2. \end{gathered} \tag{7.2}$$

Boundary conditions are

$$\begin{aligned} v = v' = \theta = 0 \quad &\text{for} \quad z = \pm 1; \\ v_p = \theta_p = 0 \quad &\text{for} \quad z = 1. \end{aligned} \tag{7.3}$$

It is assumed that perturbations of the velocity and temperature of the particle cloud vanish at the upper boundary of the layer.

The boundary-value problem (7.2), (7.3) determines the spectrum of perturbation decrements and the stability limits for an equilibrium layer of a fluid containing added particles. The Runge–Kutta–Merson stepwise method of integration is also used to solve this boundary-value problem.

§ 8. The presence of added particles shows up primarily in the spectrum of perturbation decrements. In contrast to the spectrum for a layer of pure fluid and the spectrum for a layer with transverse seepage of fluid [9, 14], the perturbation spectrum is now considerably richer because of the appearance of perturbations associated with the particle cloud. As shown by calculations, however, perturbations associated with the transport medium remain responsible for the instability of the equilibrium state.

Transverse motion of the particles leads to a considerable change in the perturbation spectrum for a stationary layer of pure fluid. Oscillational perturbations now appear in the spectrum; they arise as the result of coalescence of real levels. With an increase in Rayleigh number, these complex-conjugate pairs break down into two real levels. Instability, as in the case of a stationary layer of pure fluid, is caused by the real branches of the spectrum and has a monotonic nature.

The effect of particle settling rate on the stability of a layer is illustrated in Fig. 5, which shows the dependence of the minimum critical Rayleigh number Ra_m on the particle settling rate u_s (or, which comes to the same thing, on the Galileo number) ($\mathrm{Pr}=0.73$, $a=0.1$, $\tau_v=0.00452$, $\tau_T=0.01336$). Layer stability rises rapidly with increase in $|u_s|$. The wavelength of the most dangerous perturbations decreases. In a layer of air 2 cm thick, motion of wood particles at a velocity ≈ 20 cm/sec ($a = 0.1$, $r = 0.007$ cm) increases the stability by a factor of almost 17. With an increase in the particle settling rate, however, the rate of rise in the minimum critical Rayleigh number slows down (for $|u_s| \geq 150$).

With an increase in particle settling rate, a thermal boundary layer begins to form at the lower boundary of the layer ("blowup" of the gas temperature distribution occurs). As a result, the effective thickness of the stratified layer of gas is decreased ($h_{eff} < h$). The characteristic temperature difference of 2Θ remains fixed in this case. The critical temperature difference is found from the condition $(1 + a) g\beta\Theta h_{eff}^3/\nu\chi = \text{const}$, and therefore the critical Rayleigh number, which is determined in the usual manner from the halfwidth h of the layer, is increased in proportion to the decrease in h_{eff}, i.e., to the rise in $|u_s|$. This occurs as long as the particles which "blowup" the distribution of gas temperature increase the thickness of the thermal boundary layer at the lower surface. It turns out that at high values of the settling rate, further increase leads to insignificant distortion of the established distribution of gas temperature and so to a small rise in stabilizing effect.

Intensification of the distorting effect of particles on the distribution of gas temperature is also observed when there is an increase in the mass concentration a of the additive. The stabilizing effect of the particles on equilibrium stability increases in this case. With an increase in the mass concentration a by a factor of two from 0.1 to 0.2, the minimum critical Rayleigh number increases from 770 to 1980 and the critical wave number k_m increases from 2.19 to 2.77.

Figure 6 shows the dependence of the minimum critical Rayleigh number on the Prandtl number and on the relative heat capacity b of the particles ($a = 0.1$, $Ga/Pr = 43{,}600$, $\tau_v = 0.00452$). The curve for $Ra_m = Ra_m(Pr)$ was plotted for $b = 2.7$ and the curve for $Ra_m = Ra_m(b)$ was plotted for $Pr = 1$. With an increase in Prandtl number ($10^{-2} \leq Pr \leq 10^2$), there is a reduction in Ra_m by a factor of more than two ($Ra_m \approx 2000$ for $Pr = 0.1$ and $Ra_m \approx 1000$ for $Pr = 6$). However, convective equilibrium stability in our case is much higher than the stability of a pure fluid. Stability rises with an increase in the relative heat capacity b of the particles. Particles having a higher heat capacity better absorb the thermal perturbations that are the most dangerous.

The behavior of the minimum critical Rayleigh number Ra_m as a function of particle radius (or of relaxation time τ_v) is similar to the behavior of the minimum critical Grashof number in the problem of the stability of convective flow in a medium containing an additive in a vertical layer (see Fig. 4). An increase in r leads to an increase in equilibrium stability up to some limiting value $r_* = 0.004$ at which $Ra_m = 3125$ ($a = 0.1$, $Ga = 31{,}830$, $Pr = 0.73$, $b = 2.7$, $\rho_1/\rho = 415$). The stabilizing effect decreases when $r > r_*$. The critical wave number k_m increases with increase in r and reaches a value of 3.1 (for $r \approx 0.004$ and the given values of the problem parameters), and then decreases with further increase in r. In contrast to the problem of the stability of steady-state convective motion of a medium containing an additive (see Fig. 4), the increase in convective equilibrium stability with increase in particle size is associated with a decrease in the length of dangerous standing perturbations.

One should note in conclusion that the effect of settling particles on convective equilibrium stability in a horizontal layer of fluid is similar in many respects to the effect of transverse seepage of fluid [9, 14, 15].

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HEAT EXCHANGE BETWEEN A SELECTIVELY EMITTING LIQUID AND A LAMINAR GAS FLOW IN THE PRESENCE OF AN EXTERNAL SOURCE OF RADIATION

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An investigation is conducted in the solution of a number of practical problems of the radiative and combined heat exchange in nonuniform systems having widely different physical properties. The processes of thermal interaction between the ocean and the atmosphere have been treated in the paper [1], the effect of thermal radiation on the melting and solidification of semitransparent crystals has been investigated in [2], the flow of a selectively emitting gas around the lateral surface of an object evaporating under the action of radiative heating has been discussed in [3], and heat transfer from a jet to the molten mass of glass in a glassmaking furnace tank has been investigated in [4]. The radiative and combined heat exchange between a selectively emitting liquid and a transparent heat-conducting laminar gas flow in the case of a specified external thermal radiation field is discussed in this paper. The energy conservation equations are set up taking into account the heat transfer by radiation, convection, and molecular thermal conduction. A differential approximation is used in calculating the values of the radiation fluxes. A system of fundamental computational equations is solved by the method of finite differences and iterations and by the Runge–Kutta method. The results of the calculations are presented in the form of graphs.

CONVENTIONAL NOTATION

$\mathrm{Bo} = d\rho c_p/\sigma(T_0 - T_m)^3$ is the Boltzmann number; $\mathrm{Iw} = \sigma(T_0 - T_m)^3 a/\lambda$ is the Ivanov number; $\mathrm{Bu}_\lambda = \varkappa_\lambda a$ is the Bouguer number; $\mathrm{Re} = da/\nu$ is the Reynolds number; $\mathrm{Bi} = \alpha a/\lambda$ is the Biot number; $\theta = (T - T_m)/(T_0 - T_m)$ is the dimensionless temperature; U and V are the longitudinal and transverse dimensionless velocity components, respectively; U_0 is the dimensionless velocity of the unperturbed gas flow; $P = p/\rho d^2$ is the dimensionless pressure; $\mu = \mu_T/\mu_{T_0}$ is the dimensionless dynamic viscosity coefficient; $E_{T,\lambda} = E^0_{T,\lambda}/\sigma(T_0 - T_m)^4$ is the dimensionless energy density of the radiation from an absolutely black body; q_λ is the dimensionless radiation flux; q_T is the dimensionless flux of heat transported by conduction; $q_n = q_T + q$ is the dimensionless net heat

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